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Low-Speed Flutter and Its Physical Interpretation

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ABSTRACT

The phenomenon of low-speed flutter of an oscillating airfoil is investigated. The problem resolves itself into the study of two phenomena: that of zero air-speed classic flutter and that of an oscillating airfoil with no vortex shed. It is seen that both cases involve oscillation of the airfoil about a nodal line located three-quarters of the chord aft of the leading edge. The study is based on the fundamental theory developed by Theodorsen.

The relationship between the parameters required for zero-speed flutter for the general case is developed, and the physical interpretation of this phenomenon is discussed.

For the latter case, the solution of the equations of motion, based on noncirculatory flow, and the equations resulting from the Kutta condition result in a relationship between the inertia and elastic parameters for which the airfoil will flutter (i.e., maintain oscillations at constant amplitude) for all air speeds. The physical interpretation and the mechanism of this phenomenon are discussed.

Practical implications, such as the importance of nodal-line location and the effect of the physical parameters of the airfoil on nodal line location, are discussed.

The possibility of a new approach to the flutter problem is suggested—that of employing the ground vibration modes of the airplane to study the existence of a low flutter speed and as a design guide to raise the flutter speed.

Conclusions are also drawn regarding the influence of aspect ratio and the relative importance of theoretical aerodynamic coefficients against values of these quantities measured in the wind tunnel.

NOTATION

h	= downward displacement of airfoil at elastic axis
α	= pitch of airfoil (nose upward)
V	= air speed
ω	= flutter frequency (rad. per sec.)
I_α	= moment of inertia of airfoil about elastic axis
M	= mass of airfoil
S_α	= static moment of airfoil about the elastic axis
$\bar{I}_\alpha$	= $I_\alpha + \pi\rho b^4(^1/_8 + a^2)$ = moment of inertia of airfoil including apparent mass effect
$\bar{M}$	= $M + \pi\rho b^2$ = mass of airfoil plus apparent mass
$\bar{S}_\alpha$	= $S_\alpha - a\pi\rho b^3$ = static moment including apparent mass effect
ω_h	= natural frequency in h degree of freedom
ω_α	= natural frequency in α degree of freedom
b	= semichord
ab	= elastic axis location aft of mid-chord
k	= $\omega b/V$ = reduced frequency
ρ	= air density
$R_{a\alpha}, I_{a\alpha}$, etc.	= real and imaginary parts of aerodynamic coefficients, not including apparent mass effects
g	= structural damping

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INTRODUCTION

MUCH HAS BEEN DONE to improve the methods of flutter analysis during the last few years, but no tools have been devised which can be applied with reliance during the early stages of design. Various "tricks of the trade" are resorted to (such as the set rule of moving the center of gravity forward), but the reminder that there have been exceptions to these rules compels one to search further for more reliable guides.

With regard to flutter, the only controls the designer has are those that pertain to mass and elasticity distribution. However, the two are interdependent and the influence of the change of one on the other and the joint effect on the flutter characteristics are too complex to allow for the determination of suitable design guides begotten from intuition alone.

By examining the basic two-dimensional flutter theory of Theodorsen,[1] one becomes aware of a feature of the aircraft which incorporates both the inertia and elastic characteristics. This feature, the so-called nodal-line location, is shown to be the primary factor in the occurrence of low-speed flutter. In fact, it is shown that for an idealized airfoil the location of the nodal line at the three-quarter chord results in a zero flutter speed.

NODAL LINE

If an external vibratory load is applied to the wing of an airplane on the ground, at each of certain frequencies of excitation maximum amplitudes of wing response would be induced. It would be seen that at each of these frequencies every point of the system would move in phase with every other point and that the motion of any section, say B of Fig. 1, would be essentially the rotation of the section about a point that remains stationary in space. The locus of these nodal points is the nodal line.

Let us examine the behavior of one of these wing sections (Fig. 2). We assume that the section is free to oscillate in only two degrees of freedom. The first resonant condition will involve oscillations about a nodal point a large distance forward of the airfoil; the second will involve oscillations about a nodal point on

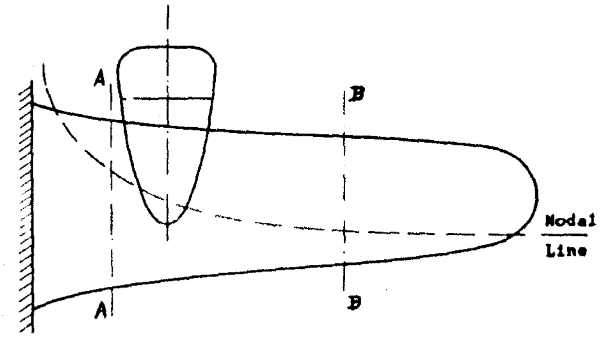

FIG. 1. Plan form of wing and nodal line.

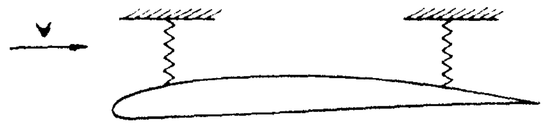

FIG. 2. Isolated section of the wing.

or near the airfoil. It is the latter point that is important in the occurrence of low-speed wing flutter.

Zero Air-Speed Flutter

Let us consider the effect of an air stream flowing over the airfoil of Fig. 2. At any air speed below the critical flutter speed, any small disturbance of the airfoil from its static equilibrium position will result in an oscillatory motion that will damp out with time. As the flutter speed is approached, the rate of decay diminishes until at the critical flutter speed there is no decay and the airfoil maintains a constant amplitude oscillation. At this critical speed there is no exchange of energy between the mechanical system and the stream. At any speed above that of flutter, the mechanical system absorbs energy resulting in oscillations whose amplitude builds up with time.

Consider now the airfoil of Fig. 2 in still air. If we neglect the structural and air friction, any displacement from equilibrium will result in constant amplitude oscillations. However, this phenomenon will be a limiting case of flutter for only special distributions of mass and elasticity of the system. It will now be shown that the necessary and sufficient condition for zero air-speed flutter is that the mass and elasticity of the system be such as to result in a node at the three-quarter chord aft of the leading edge.

The equations of motion of an airfoil oscillating in a uniform stream are:

$$-\bar{I}_\alpha(\omega^2 - \omega_\alpha^2)\alpha - \omega^2\bar{S}_\alpha h = -\pi\rho b^4\omega^2[(R_{\alpha\alpha} + iI_{\alpha\alpha})\alpha + (R_{\alpha h} + iI_{\alpha h})(h/b)] \quad (1)$$

$$-\omega^2\bar{S}_\alpha\alpha - \bar{M}(\omega^2 - \omega_h^2)h = -\pi\rho b^3\omega^2[(R_{c\alpha} + iI_{c\alpha})\alpha + (R_{ch} + iI_{ch})(h/b)] \quad (2)$$

where ω_h and ω_α are the natural frequencies of the system in still air, the former for pure bending and the latter for pure torsion. $\bar{I}_\alpha$, $\bar{S}_\alpha$, and $\bar{M}$ are the moment of inertia and static moment about the elastic axis, and mass, respectively, all including apparent mass contributions. In nondimensional form, Eqs. (1) and (2) become

$$\left[-\frac{\bar{I}_\alpha}{\pi\rho b^4}\left(1 - \frac{\omega_\alpha^2}{\omega^2}\right) + (R_{\alpha\alpha} + iI_{\alpha\alpha})\right]\alpha + \left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + (R_{\alpha h} + iI_{\alpha h})\right]\frac{h}{b} = 0 \quad (3)$$

$$\left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + (R_{c\alpha} + iI_{c\alpha})\right]\alpha + \left[-\frac{\bar{M}}{\pi\rho b^2}\left(1 - \frac{\omega_h^2}{\omega^2}\right) + (R_{ch} + iI_{ch})\right]\frac{h}{b} = 0 \quad (4)$$

The stability determinant becomes

$$\begin{vmatrix} \left[-\frac{\bar{I}_\alpha}{\pi\rho b^4}\left(1 - \frac{\omega_\alpha^2}{\omega^2}\right) + R_{\alpha\alpha} + i\frac{1}{k}I_{\alpha\alpha}'\right] & \left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + R_{\alpha h} + i\frac{1}{k}I_{\alpha h}'\right] \\ \left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + R_{c\alpha} + i\frac{1}{k}I_{c\alpha}'\right] & \left[-\frac{\bar{M}}{\pi\rho b^2}\left(1 - \frac{\omega_h^2}{\omega^2}\right) + R_{ch} + i\frac{1}{k}I_{ch}'\right] \end{vmatrix} = 0 \quad (5)$$

where $I_{\alpha\alpha} \doteq \frac{1}{k}I_{\alpha\alpha}'$, etc.

Separating into real and imaginary parts,

$$\begin{vmatrix} \left[-\frac{\bar{I}_\alpha}{\pi\rho b^4}\left(1 - \frac{\omega_\alpha^2}{\omega^2}\right) + R_{\alpha\alpha}\right] & \left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + R_{\alpha h}\right] \\ \left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + R_{c\alpha}\right] & \left[-\frac{\bar{M}}{\pi\rho b^2}\left(1 - \frac{\omega_h^2}{\omega^2}\right) + R_{ch}\right] \end{vmatrix} - \frac{1}{k^2}\begin{vmatrix} I_{\alpha\alpha}' & I_{\alpha h}' \\ I_{c\alpha}' & I_{ch}' \end{vmatrix} = 0 \quad (6)$$

$$\frac{1}{k}\left\{\begin{vmatrix} \left[-\dfrac{\bar{I}_\alpha}{\pi\rho b^4}\left(1-\dfrac{\omega_\alpha^2}{\omega^2}\right)+R_{a\alpha}\right] & I_{ah}' \\ \left[-\dfrac{\bar{S}_\alpha}{\pi\rho b^3}+R_{c\alpha}\right] & I_{ch}' \end{vmatrix} + \begin{vmatrix} I_{a\alpha}' & \left[-\dfrac{\bar{S}_\alpha}{\pi\rho b^3}+R_{ah}\right] \\ I_{c\alpha}' & \left[-\dfrac{\bar{M}}{\pi\rho b^2}\left(1-\dfrac{\omega_h^2}{\omega^2}\right)+R_{ch}\right] \end{vmatrix}\right\} = 0 \qquad (7)$$

The factor $1/k$ of Eq. (7), put equal to zero combined with Eq. (6), is the condition for steady-state oscillations at zero air speed but not the condition for flutter. It is the simultaneous vanishing of Eq. (6) and the second factor of Eq. (7) which gives the flutter solution. At zero air speed

$$1/k = R_{a\alpha} = R_{ah} = R_{c\alpha} = R_{ch} = 0$$

and

$$I_{a\alpha}' = (1/2 - a)^2, \qquad I_{ah}' = -(1/2 + a)$$

$$I_{c\alpha}' = 3/2 - a, \qquad I_{ch}' = 1$$

Substituting, Eqs. (6) and (7) become,

$$\begin{vmatrix} \dfrac{\bar{I}_\alpha}{\pi\rho b^4}\left(1-\dfrac{\omega_\alpha^2}{\omega^2}\right) & \dfrac{\bar{S}_\alpha}{\pi\rho b^3} \\ \dfrac{\bar{S}_\alpha}{\pi\rho b^3} & \dfrac{\bar{M}}{\pi\rho b^2}\left(1-\dfrac{\omega_h^2}{\omega^2}\right) \end{vmatrix} = 0 \qquad (8)$$

and

$$\begin{vmatrix} \dfrac{\bar{I}_\alpha}{\pi\rho b^4}\left(1-\dfrac{\omega_\alpha^2}{\omega^2}\right) & (1/2 - a) \\ \dfrac{\bar{S}_\alpha}{\pi\rho b^3} & -1 \end{vmatrix} - \begin{vmatrix} (1/2 - a)^2 & \dfrac{\bar{S}_\alpha}{\pi\rho b^3} \\ (3/2 - a) & \dfrac{\bar{M}}{\pi\rho b^2}\left(1-\dfrac{\omega_h^2}{\omega^2}\right) \end{vmatrix} = 0 \qquad (9)$$

Eq. (8) is the usual zero air-speed frequency equation. The important equation is Eq. (9). Expanding it, we obtain

$$\left(\frac{1}{2} - a\right)^2 b^2\bar{M}\left(1-\frac{\omega_h^2}{\omega^2}\right) + \bar{I}_\alpha\left(1-\frac{\omega_\alpha^2}{\omega^2}\right) + (2a - 1)b\bar{S}_\alpha + \frac{3}{8}\pi\rho b^4 = 0 \qquad (10)$$

which is the equation governing the oscillating motion of an airfoil elastically restrained at the elastic axis but constrained by a node at the three-quarter chord point as shown in Fig. 3.

The total restoring moment for the system of Fig. 3 about the three-quarter chord is

$$M_R = [b^2(1/2 - a)^2\omega_h^2\bar{M} + \omega_\alpha^2\bar{I}_\alpha]\alpha \qquad (11)$$

The moment of the inertia forces about the three-quarter chord is

$$M_I = [\bar{I}_\alpha + \bar{M}(1/2 - a)^2 b^2 + 2b(a - 1/2)\bar{S}_\alpha]\ddot{\alpha} \qquad (12)$$

The aerodynamic moment about the three-quarter chord point when $1/k = 0$ is

$$M_\alpha = -(3/8)\pi\rho b^4 \qquad (13)$$

We have the dynamical equation

$$M_R + M_I = M_\alpha$$

For harmonic motion this equation becomes

$$\left(\frac{1}{2} - a\right)^2 b^2\bar{M}\left(1-\frac{\omega_h^2}{\omega^2}\right) + \bar{I}_\alpha\left(1-\frac{\omega_\alpha^2}{\omega^2}\right) + (2a - 1)b\bar{S}_\alpha + \frac{3}{8}\pi\rho b^4 = 0$$

which is identical with Eq. (10), the condition for zero-speed flutter.

We see then that the condition for zero air-speed flutter is that the nodal line is at the three-quarter chord.

The importance of the nodal-line location in flutter is then apparent. One should therefore investigate the variation of flutter speed with nodal-line location as one of the parameters. This has been done for various combinations of parameter values. When the structural damping g is not zero, the flutter speed does not vanish for a node location at the three-quarter chord; however, it reaches a minimum in the vicinity of that location. The variation of flutter speed with nodal-line location is illustrated in Fig. 4 for two values of the structural damping.

Vortex-Free Flutter

There is a case of occurrence of zero air-speed flutter in a two-dimensional airfoil for which flutter occurs also at all other speeds. This type of flutter is as-

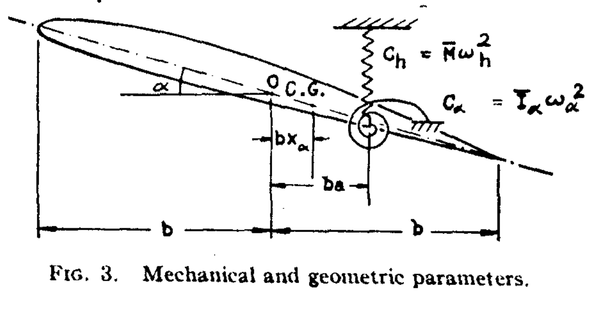

Fig. 3. Mechanical and geometric parameters.

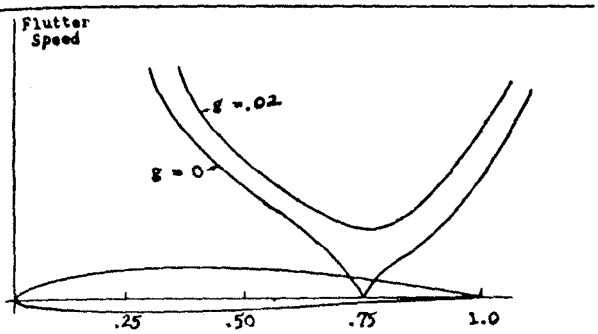

FIG. 4. Flutter speed versus nodal line location (node location in chord lengths).

sociated with the vanishing of the circulation about the airfoil. The aerodynamic force and moment experienced by an oscillating airfoil is made up of two parts—a noncirculatory and a circulatory. If the latter, which also is associated with the shedding of vorticity from the trailing edge of the airfoil, is assumed to vanish, the equations of motion become

$$\ddot{\alpha}\bar{I}_\alpha + \alpha\omega_\alpha{}^2\bar{I}_\alpha + \bar{S}_\alpha\ddot{h} + \pi\rho b^4\omega^2\left[-\frac{1}{k}\left(\frac{\alpha}{k} + \frac{h}{\omega b}\right)\right] = 0 \tag{14}$$

$$\ddot{h}\bar{M} + h\omega_h{}^2\bar{M} + \bar{S}_\alpha\ddot{\alpha} + \pi\rho b^3\omega^2\left[\left(\frac{1}{k}\right)\left(\frac{\dot{\alpha}}{\omega}\right)\right] = 0 \tag{15}$$

For harmonic motion, in nondimensional form, Eqs. (14) and (15) become

$$\left[-\left(1 - \frac{\omega_\alpha{}^2}{\omega^2}\right)\frac{\bar{I}_\alpha}{\pi\rho b^4} - \frac{1}{k^2}\right] + \left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} - i\left(\frac{1}{k}\right)\left(\frac{h}{b}\right)\right] = 0 \tag{16}$$

$$\left[-\frac{\bar{S}_\alpha}{\pi\rho b^3} + i\left(\frac{1}{k}\right)\right]\alpha + \left[-\frac{\bar{M}}{\pi\rho b^2}\left(1 - \frac{\omega_h{}^2}{\omega^2}\right)\right]\frac{h}{b} = 0 \tag{17}$$

Since there is no circulation about the airfoil, the Kutta condition becomes

$$V\alpha + \dot{h} + b(1/2 - a)\dot{\alpha} = 0 \tag{18}$$

In nondimensional form

$$\left[(1/2 - a) - i\left(\frac{1}{k}\right)\right]\alpha + [1]\frac{h}{b} = 0 \tag{19}$$

From Eqs. (16) and (17),

$$\begin{vmatrix} +[X + (1/k^2)] & [Z + i(1/k)] \\ [Z - i(1/k)] & Y \end{vmatrix} = 0 \tag{20}$$

and from Eqs. (17) and (19)

$$\begin{vmatrix} (1/2 - a) - i(1/k) & 1 \\ [Z - i(1/k)] & Y \end{vmatrix} = 0 \tag{21}$$

where

$$X = (\bar{I}_\alpha/\pi\rho b^4)\,[1 - (\omega_\alpha{}^2/\omega^2)]$$
$$Y = (\bar{M}/\pi\rho b^2)\,[1 - (\omega_h{}^2/\omega^2)]$$
$$Z = \bar{S}_\alpha/\pi\rho b^3$$

Eq. (20) becomes

$$Y[X + (1/k^2)] - [Z^2 + (1/k^2)] = 0 \tag{22}$$

while Eq. (21) becomes:

$$\left.\begin{aligned} (1/2 - a)Y - Z &= 0 \\ (1/k)(1 - Y) &= 0 \end{aligned}\right\} \tag{23}$$

Eqs. (23) are identically satisfied for all values of $1/k$ if

$$\left.\begin{aligned} Y &= 1 \\ (1/2) &- a \end{aligned}\right\} \tag{24}$$

Substituting Eq. (24) into the determinant, Eq. (20),

$$X = (1/2 - a)^2 \tag{25}$$

as the condition for flutter. Since Eq. (25) is independent of $1/k$, it is seen that this type of instability will persist at all air speeds—i.e., flutter will occur at any speed from 0 to ∞.

From Eq. (19) it is seen that, when $1/k = 0$ ($V = 0$),

$$h/b + (1/2 - a)\alpha = 0 \tag{26}$$

which is the condition that the nodal point be at the three-quarter chord. A further examination of the Kutta condition, Eq. (18), reveals that the motion of the airfoil during this vortexless flutter is one of sinusoidal gliding at the three-quarter chord point as portrayed in Fig. 5.

We have then the following conditions for vortexless flutter:

$$X = \frac{\bar{I}_\alpha}{\pi\rho b^4}\left(1 - \frac{\omega_\alpha{}^2}{\omega^2}\right) = \left(\frac{1}{2} - a\right)^2 \tag{i}$$

$$Y = \frac{\bar{M}}{\pi\rho b^2}\left(1 - \frac{\omega_h{}^2}{\omega^2}\right) = 1 \tag{ii}$$

$$Z = \frac{\bar{S}_\alpha}{\pi\rho b^3} = \frac{S_\alpha}{\pi\rho b^3} - a = \frac{1}{2} - a \tag{iii}$$

Node at three-quarter chord (iv)

However, conditions (iv) and (iii) imply Eqs. (i) and (ii); therefore, the necessary and sufficient conditions for vortex-free flutter are, from Eqs. (iii) and (iv), that

FIG. 5. Motion of the wing in vortex-free flutter.

the node is at the three-quarter chord point and that the static moment about the elastic axis is a small value determined by

$$S_\alpha = {}^1/_2\pi\rho b^3 \tag{27}$$

The frequency of flutter ω is independent of the velocity, and the frequency ratio ω_h/ω_α is near unity. Therefore, when, in addition to a three-quarter chord location of the nodal line, the frequency ratio is near unity, the flutter speed will be extremely sensitive to changes in the frequency ratio.

Three-Dimensional Considerations

We must realize that a wing may be assumed to be composed of a continuous distribution of two-dimensional sections of the type we have been describing. Each of these sections would have its own flutter characteristics if it were separated from the rest of the wing and placed in a two-dimensional strteam. For example, section A of Fig. 1, whose nodal line is at 40 per cent of the chord, would have a high flutter speed, but section B, whose nodal line is close to the three-quarter chord, would have a low flutter speed. At a given speed, section A would be dissipating energy to the stream, while section B would be absorbing energy from the stream. At the air speed at which the amount of energy lost by such sections as A equals that absorbed by those such as B, flutter occurs. The obvious guide is to design the wing so that the nodal line is as far from the three-quarter chord as is practicable at all spanwise stations of the wing.

During the early stages of design, the nodal line and the effect of various design changes on its location can be determined analytically.

Another important consequence of the present point of view is the tendency of vortex-free flutter to be independent of aspect ratio. It is to be expected that certain types of flutter occur at or near the vortex-free régime. In this case, correction of three-dimensional aerodynamic effects will have little influence on the flutter speed.

It should also be noted that vortex-free flutter involves only perfect fluid theory and does not introduce the action of viscosity since no circulation is produced. This is probably a contributing factor to the remarkable experimental verification of the theory which has been frequently observed. It may well be that the substitution of experimental aerodynamic coefficients measured on oscillating models can lead to less reliable predictions of flutter in certain cases.

Conclusions

(a) It is recommended that a study of locations of nodal lines be employed to ensure against the occurrence of flutter.

(b) The designer should attempt to locate the nodal line as far from the three-quarter chord as is practicable.

(c) A limiting case of flutter without vorticity may occur. This may account for the small effect that aspect ratio corrections have had on flutter in many cases studied.

(d) The importance of the theoretical aerodynamic coefficients should be emphasized, since it is entirely possible that in certain types of flutter they may lead to more reliable predictions of the flutter speed than the experimental coefficients.

Reference

[1] Theodorsen, T., *General Theory of Aerodynamic Instability and the Mechanism of Flutter*, N.A.C.A. T.R. No. 496, 1940.