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On the Reflection of Acoustic Waves on a Rough Surface

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IN a previous paper[1] we derived closed form expressions for the reflection on a rough surface of acoustic waves generated by a simple harmonic point source. Because of an error in sign for σ the problem which has been solved in the above paper corresponds to acoustic waves in a liquid being reflected on a perfectly rigid smooth surface covered with small air bubbles. This is easily seen if we remember that the dipole induced in the air bubble is of opposite sign to that induced in a solid protuberance. Quantitatively in this case we must also replace $\frac{1}{2}\sigma$ by σ and the value of κ by $\kappa=1-(\pi^2/2)(a^3/b^3)$.

The actual problem intended in reference 1 is that of reflection on a rigid surface with rigid bosses. In this case the boundary condition (2.18) of reference 1 must undergo a change in sign and read

$$\frac{\partial\phi}{\partial z}=-\tfrac{1}{2}\sigma\left(\frac{\partial^2}{\partial z^2}+k^2\right)\phi \tag{1}$$

$$\sigma=2\pi Na^3/\kappa. \tag{2}$$

A plane wave when reflected on the rough surface is multiplied by $\exp(2i\psi)$ where the phase angle ψ is given by

$$\tan\psi=-\tfrac{1}{2}\sigma k\frac{\sin^2\theta}{\cos\theta} \tag{3}$$

with θ the angle of incidence. Except for the change in sign this phase angle is the same as given by Eq. (5.17) of reference 1.

An interesting feature resulting from the change in sign is the appearance of a surface wave. This corresponds to a solution

$$\phi=e^{-z\beta}e^{-ilx} \tag{4}$$

where β is real positive. It must satisfy the wave equation and the boundary condition (1). Hence

$$\begin{aligned}\beta^2+k^2&=l^2\\ \beta&=\tfrac{1}{2}\sigma(\beta^2+k^2)\end{aligned} \tag{5}$$

or

$$\begin{aligned}\beta&=\tfrac{1}{2}\sigma l^2\\ k^2&=l^2(1-\tfrac{1}{4}\sigma^2l^2).\end{aligned} \tag{6}$$

For the theory to be valid the wavelength along the surface must be large compared to the size of the roughness. Referring to the definition of σ this means $\sigma l<\frac{1}{2}$. For any given value $k^2<(1/\sigma^2)$ the second Eq. (6) has two real roots for l^2. However the largest of the two must be dropped because it corresponds to a wavelength which is not large enough relative to the roughness for the theory to be applicable. Hence for a given frequency there is only one surface wave. The phase velocity of the surface wave is

$$v=kc/l=c(1-\tfrac{1}{4}\sigma^2l^2)^{\frac{1}{2}}. \tag{7}$$

Since $\sigma l<\frac{1}{2}$, there is a slight dispersion.

The reflected waves due to a point source are obtained by changing σ to $-\sigma$ in expression (3.9) of reference 1. The velocity potential ϕ_r of the reflected field is

$$\phi_r=D\int_0^\infty\frac{1}{\mu}\frac{\mu+\frac{1}{2}\sigma l^2}{\mu-\frac{1}{2}\sigma l^2}J_0(lr)e^{-\mu(z+h)}l\,dl. \tag{8}$$

The change in sign introduces real roots for l in the denominator. These poles correspond to the surface waves generated by the point source. The solution of the integral has to be modified accordingly.

[1] M. A. Biot, J. Acoust. Soc. Am. **29**, 1193–1200 (1957).