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On the Reflection of Electromagnetic Waves on a Rough Surface

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IN a previous publication[1] we have analyzed the reflection of a plane electromagnetic wave on a rough surface. Due to an error in sign* in Eq. (3.6) of the foregoing reference the sign on the right side of Eqs. (3.7) and (3.9) must also be changed. This amounts to replacing the "roughness parameter" σ by $-\sigma$ throughout the paper. In particular the boundary condition (7.8) for the case of vertical polarization should read

$$\frac{\partial V_t}{\partial z}=-\frac{\sigma}{\kappa_1}\left[\frac{\partial^2}{\partial z^2}+k^2\left(1-\frac{1}{2}\frac{\kappa_1}{\kappa_2}\right)\right]V_t \qquad (1)$$

$$(\sigma=2nNa^3).$$

The phase angle 2φ of the reflected wave as a function of the angle of incidence α is

$$\tan\varphi=\tfrac{1}{2}\sigma\frac{k}{\kappa_1}\frac{\cos 2\alpha+\frac{\kappa_1}{\kappa_2}-1}{\cos\alpha}. \qquad (2)$$

In the diagram of Fig. 6 (reference 1) showing φ *versus* α the sign of φ must be reversed. Because of this change in sign at normal incidence the effect of the roughness is to raise the effective surface of reflection contrary to conclusion of the paper. The vanishing influence of the roughness near an incidence of 45 degrees for a perfect conductor and the 180 degree phase change at grazing incidence are not affected.

An interesting feature which is added by the change in sign is the appearance of a surface wave. For such a wave to exist there must be a solution of the type

$$V_t=e^{-\beta z}e^{-ilx}. \qquad (3)$$

It must satisfy the wave equation and the boundary condition (1). Hence we must have simultaneously

$$\beta=\frac{\sigma}{\kappa_1}\left[\beta^2+k^2\left(1-\frac{1}{2}\frac{\kappa_1}{\kappa_2}\right)\right] \qquad (4)$$

$$\beta^2+k^2=l^2.$$

The values of β, k^2, and l^2 must be real and positive. Moreover for the theory to be valid the wavelength cannot be of the order of the size of the roughness, say about five times the roughness size. This last condition may be written

$$\sigma l<\tfrac{1}{2}. \qquad (5)$$

Elimination of k^2 in Eqs. (4) yields

$$\frac{\sigma^2\beta^2}{\kappa_2}-2\beta\sigma+\sigma^2l^2\left[\frac{2}{\kappa_1}-\frac{1}{\kappa_2}\right]=0. \qquad (6)$$

There are two roots β_1 and β_2 of this equation

$$\begin{aligned}\frac{1}{\kappa_2}\sigma\beta_1&=1+\left[1-\frac{1}{\kappa_2}\left(\frac{2}{\kappa_1}-\frac{1}{\kappa_2}\right)\sigma^2l^2\right]^{\frac{1}{2}}\\ \frac{1}{\kappa_2}\sigma\beta_2&=1-\left[1-\frac{1}{\kappa_2}\left(\frac{2}{\kappa_1}-\frac{1}{\kappa_2}\right)\sigma^2l^2\right]^{\frac{1}{2}}.\end{aligned} \qquad (7)$$

Due to condition (5) and the fact that κ_1 and κ_2 are near unity the roots are real and positive. However the first root is such that

$$\sigma\beta_1>1 \qquad (8)$$

which combined with condition (5) and the second Eq. (4) leads to an imaginary value for k. Hence the root β_1 is not physically valid. By expansion of the radical to the first order term in σ^2l^2 we find

$$\beta_2=\sigma l^2\left[\frac{1}{\kappa_1}-\frac{1}{2\kappa_2}\right]. \qquad (9)$$

This corresponds to a surface wave. Using the approximate value (9) for β we derive

$$k=l\left[1-\sigma^2l^2\left(\frac{1}{\kappa_1}-\frac{1}{2\kappa_2}\right)^2\right]^{\frac{1}{2}}. \qquad (10)$$

This leads to a slightly dispersive phase velocity

$$v=\frac{kc}{l}=c\left[1-\sigma^2l^2\left(\frac{1}{\kappa_1}-\frac{1}{2\kappa_2}\right)^2\right]^{\frac{1}{2}}. \qquad (11)$$

If we examine the case of horizontal polarization we find a single positive root β but it is too large to correspond to the condition that the wavelength is small relative to the roughness size. There is therefore no horizontally polarized surface wave within the scope of the present theory.

The case where no magnetic dipoles are induced amounts to putting $\kappa_2=\infty$ leading again to a surface wave. The acoustic case corresponds to $\kappa_1=2\kappa$ and $\kappa_2=\infty$ where κ is the same factor as κ_1. This leads also to the existence of an acoustic surface wave.

[1] M. A. Biot, J. Appl. Phys. **28**, 1455–1463 (1957).
* The error in sign was called to the authors attention by S. P. Morgan of Bell Telephone Laboratories.