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Stability Problems of Inhomogeneous Viscoelastic Media

by

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The phenomenon of buckling in compression of an elastic inhomogeneous medium under initial stress is well known. It is illustrated by the instability of a longitudinally compressed elastic plate embedded in a medium of lower rigidity. Under a critical load buckling appears suddenly in the form of a sinusoidal folding of a certain wave-length. Such sudden buckling will, of course, not occur in general if the solids involved are viscoelastic.

The interest in studying the stability problem of viscoelastic media lies partly in some technological applications connected with the use of polymers or with the thermomechanics of metallic structures. Our main interest, however, was directed to geodynamics in an attempt to furnish the beginnings of a quantitative approach to problems of deformation of the earth's crust. A systematic program to this effect was initiated about ten years ago and embodies several distinct areas of investigation. One such investigation was the development of general stress-strain relations derived from the thermodynamics of irreversible processes [1]. We also had to establish a theoretical approach for the treatment of stability problems in viscoelastic media [2]. This was applied to the gradual clarification of the qualitative as well as quantitative nature of the phenomena. Along with the theoretical work a model testing program has also been initiated as an intermediate check toward its application to full-scale geophysical configurations.

The simplest problem is that of a layer embedded in an infinite medium or lying on the surface of a half-space. A longitudinal compression acts in the layer. This problem has been treated quite simply by analogy with the corresponding elastic problem [3]. The stress-strain relations are formulated operationally in the same form as in the elastic case, but the elastic moduli are replaced by operators in conformity with a general "correspondence rule". The nature of these operators was derived from the thermodynamics of irreversible processes and Onsager's relations. The correspondence rule leads to an immediate solution by using the known equations for the elastic case. It is found that, in general, instability occurs for a range of wave-lengths. There is a wave-length whose amplitude increases at the fastest rate, which we have referred to as the "dominant wave-length". Simple expressions can be established for this dominant wave-length for the general case. In particular, if the solids are purely viscous with a viscosity μ for the layer of thickness h and a viscosity μ_1 for the surrounding medium, the dominant wave-lentgh is independent of the compression and its value is found to be

$$L_d = 2\pi h \sqrt[3]{\mu/(6\mu_1)}.$$

For the more general case of two viscoelastic media the dominant wave-length depends on the compression and on the ratio of relaxation times of the two media. The influence of adherence between the layer and the medium was investigated and found to be small.

In addition to the dominant wave-length another important feature is the amplitude of folding. For the instability to be significant the rate of growth of the folding must be sufficiently high as compared to the over-all compression rate. The theory shows that, in the case of purely viscous solids, this requires the viscosity of the layer to be at least of the order of sixty times that of the surrounding medium.

The next phase involved is the development of a rigorous theory which treats the layer as a two-dimensional continuum instead of using the approximate plate theory. Furthermore, the influence of the initial stress in the surrounding medium cannot rigorously be neglected. The rigorous treatment was accomplished by applying equations developed earlier by this writer in a series of pre-war publications for an elastic continuum under initial stress. The equations are immediately applicable to viscoelastic media. Results indicate the limits of applicability of the previous approximate results. As a "by-product" of this investigation we also solved the problem of stability of a viscoelastic half-space under a compression parallel with the surface. The surface is found to be unstable. This is offered as an explanation for the wrinkles which appear at the surface of a compressed solid in the plastic range.

From the viewpoint of geodynamics, the influence of gravity on the stability is of paramount interest. This was investigated for the compressed layer at the surface of a half-space and for the layer embedded between top and bottom media of different densities and viscoelastic properties. The effect of gravity is embodied in a parameter which involves the ratio of the horizontal compression to the gravity force. In the case where the top material is denser, a spontaneous instability appears due to gravity alone and with a wave-length different from that produced by the horizontal compression. The stability of the viscoelastic half-space with properties varying continuously with depth has also been investigated.

When considering applications of the theory to problems of deformation and flow of the earth's crust, the significant fact to bear in mind is that viscosities of rock vary in much wider range than the elastic rigidities. Formulas obtained through the present theory lead to conclusions which agree with geological observations. They also lead to interesting conclusions regarding the mechanism of formation of salt domes and current theories of orogenesis.

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[3] — *Folding instability of a layered viscoelastic medium under compression*, Proc. Roy. Soc., A. **242** (1957), 444—454; (Other phases of the work reported here will appear in a sequence of forthcoming publications).