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Exact Simplified Non-Linear Stress and Fracture Analysis around Cavities in Rock

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A non-linear analysis for finite strain and stress has been developed and applied to symmetrical deformations around cylindrical and spherical cavities in rock. The material non-linearity which is involved may result from either elastic or plastic properties. A drastic analytical simplification reduces the problem to the solution of a first order differential equation. Fracture is analyzed.

1. INTRODUCTION

The mechanical properties of rock materials are in general strongly non-linear. Because of mathematical difficulties, such properties are usually not taken into account in stress analysis. However, as shown in this paper there are special cases for which such an analysis can be made extremely simple using elementary methods while retaining at the same time complete mathematical rigor. These two cases are those of axial and spherical symmetry of the field. Thus a realistic analysis becomes feasible of rock fracture around a cylindrical or spherical cavity due to injection of fluid under pressure in the cavity.

The key to the procedure as shown in section 2 in the context of axial symmetry is provided by deriving a first order ordinary differential equation for the two stress components considering one of them as a function of the other. The integrals of this equation constitute a one parameter family of curves relating the two stresses. On such a stress diagram it is also possible to represent quite simply the failure condition of the material as a single curve relating the two stresses at failure. Thus the stress analysis not only takes into account the actual non-linear properties but it leads at the same time to an analysis of failure.

This has been applied in section 3 to the problem of crack initiation due to fluid pressure in a cylindrical cavity in rock material. Two types of non-linear properties are considered. In one case the material remains nearly elastic with non linearity due to closing of the pores. In the other plastic behavior predominates.

The case of a spherical cavity is treated in section 4 by an entirely similar method. Except for a change of slope of the plots, results are completely analogous to those obtained for a cylindrical cavity.

The analysis does not take into account fluid penetration into the porous rock. This implies a sealed cavity wall or a fluid of very high viscosity.

Application of the method is not restricted to rock material and is quite general. In particular it provides also a simple and rigorous approach to the non-linear stress analysis of thick cylindrical or spherical containers of structural material.

Experimental and theoretical studies on stresses and fractures around cavities have been treated in a different way by a number of authors. Le Tirant and Baron [1] investigated the fracture of cylindrical cavities in the context of the linear theory. In soil mechanics a non-linear treatment of cylindrical and spherical cavities was provided by Vesic [2]. The present treatment shows that a very simple analytical procedure is available which can be readily interpreted graphically. In the discussion use has been made of the results of Brace [3] and Brady [4] regarding the fundamental non-linear behavior of rocks.

2. NON-LINEAR STRESS FIELD WITH AXIAL SYMMETRY

We shall consider a continuous medium undergoing a deformation with axial symmetry around a z axis. A state of plane strain is assumed which is the same in all planes normal to the z axis. During deformation a material point initially at a radial distance r from the axis is displaced along the same radial direction by an amount U, function of r only. At the displaced point the *finite strain* is represented by the principal components ϵ_1 in the radial direction and the principal component ϵ_2 in the circumferential direction. Their values are

$$\epsilon_1 = -\frac{dU}{dr} \qquad \epsilon_2 = -\frac{U}{r} \tag{2.1}$$

Note that the sign is chosen so that positive values represent compressive strains.

The corresponding principal stresses are denoted by τ_1 and τ_2 respectively in the radial and circumferential direction. They are defined as the normal forces acting at the displaced point per unit initial area of the medium. The sign of the stress is chosen positive for a compression. The equilibrium condition for this stress field is readily obtained from the principle of virtual work. Consider the virtual work of the stress field

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in a circular slab of unit thickness along the axis. This virtual work must vanish. Hence

$$2\pi \int (\tau_1 \delta\epsilon_1 + \tau_2 \delta\epsilon_2) r \, dr = 0 \tag{2.2}$$

with

$$\delta\epsilon_1 = -\frac{d}{dr}\delta U \quad \delta\epsilon_2 = -\frac{\delta U}{r} \tag{2.3}$$

The integration is performed along any range of r and within this range the virtual displacement is arbitrary. After integration by parts, condition (2.2) yields

$$\frac{d\tau_1}{dr} + \frac{\tau_1 - \tau_2}{r} = 0 \tag{2.4}$$

Note that this equilibrium equation is completely general and is valid for arbitrary large strains.

A compatibility relation for the finite strains ϵ_1 and ϵ_2 is derived by eliminating U between relations (2.1). We obtain

$$\frac{d\epsilon_2}{dr} = \frac{\epsilon_1 - \epsilon_2}{r} \tag{2.5}$$

The physical properties of the material are represented by the stress–strain relations which are written

$$\begin{aligned} \epsilon_1 &= \epsilon_1(\tau_1, \tau_2) \\ \epsilon_2 &= \epsilon_2(\tau_1, \tau_2) \end{aligned} \tag{2.6}$$

They correspond to finite deformations either elastic or plastic. The only assumption we shall introduce here is the property of isotropy in the plane of deformation. This is expressed quite simply by the condition that ϵ_2 is obtained from ϵ_1 by interchanging the variables τ_1 and τ_2. Hence

$$\epsilon_2(\tau_1, \tau_2) = \epsilon_1(\tau_2, \tau_1) \tag{2.7}$$

Note that if the medium is elastic the functions ϵ_1 and ϵ_2 must also satisfy the additional relation

$$\frac{\partial\epsilon_1}{\partial\tau_2} = \frac{\partial\epsilon_2}{\partial\tau_1} \tag{2.8}$$

This is a consequence of the fact that for an elastic material the expression

$$\tau_1 \delta\epsilon_1 + \tau_2 \delta\epsilon_2 - \delta(\tau_1\epsilon_1 + \tau_2\epsilon_2) = -\epsilon_1 \delta\tau_1 - \epsilon_2 \delta\tau_2 \tag{2.9}$$

is an exact differential.

In the present analysis the additional assumption (2.8) is not required, and the results are applicable to isotropic materials with either elastic or plastic properties.

A drastic simplification in the analysis is obtained by eliminating r between equations (2.4) and (2.5). This leads to the relation

$$\frac{d\tau_1}{d\epsilon_2} = -\frac{\tau_1 - \tau_2}{\epsilon_1 - \epsilon_2} \tag{2.10}$$

Introduction of the values (2.6) of ϵ_1 and ϵ_2 as functions of τ_1 and τ_2 yields

$$\frac{d\tau_1}{d\tau_2} = -\frac{\partial\epsilon_2/\partial\tau_2}{[(\epsilon_1 - \epsilon_2)/(\tau_1 - \tau_2)] + \partial\epsilon_2/\partial\tau_1} \tag{2.11}$$

The right side is a function of τ_1 and τ_2.

Hence relation (2.11) is a first order differential equation for τ_1 as a function of the independent variable τ_2. The general solution of the differential equation is written

$$\tau_2 = \psi(\tau_1, C) \tag{2.12}$$

which represents a one parameter family of curves with the parameter C as a constant of integration.

There remains to determine the stress field as a function of the coordinate r. Consider for example, a thick hollow cylinder, with a stress $\tau_1 = P_a$ acting at the internal boundary $r = a$ and a stress $\tau_1 = P_b$ at the external boundary $r = b$. We substitute the value (2.12) of τ_2 into the equilibrium equation (2.4). This yields

$$\frac{d\tau_1}{\psi(\tau_1, C) - \tau_1} = \frac{dr}{r} \tag{2.13}$$

and by simple quadrature

$$\int_{P_a}^{\tau_1} \frac{d\tau_1}{\psi(\tau_1, C) - \tau_1} = \log\frac{r}{a} \tag{2.14}$$

The constant of integration C is determined by the following boundary condition at $r = b$

$$\int_{P_a}^{P_b} \frac{d\tau_1}{\psi(\tau_1, C) - \tau_1} = \log\frac{b}{a} \tag{2.15}$$

When the value of C is known the stress field is expressed as a function of r by equations (2.12) and (2.14).

This of course assumes that P_a and P_b are known at the boundaries. Actually P_a and P_b are stresses per unit initial areas while usually what is given are hydrostatic fluid pressures p_a and p_b per unit deformed areas. The two sets of stresses are related by the equations

$$\begin{aligned} P_a/p_a &= 1 + \epsilon_2[P_a, \psi(P_a, C)] \\ P_b/p_b &= 1 + \epsilon_2[P_b, \psi(P_b, C)] \end{aligned} \tag{2.16}$$

where ψ is the general integral (2.12). These equations may be solved for P_a and P_b which are then expressed as

$$\begin{aligned} P_a &= P_a(C, p_a) \\ P_b &= P_b(C, p_b) \end{aligned} \tag{2.17}$$

Substitution of these values into the boundary condition (2.15) provides an equation for the unknown C.

In practice however, when ϵ_1 does not exceed a few per cent, this additional complication may be avoided by using the approximation

$$\begin{aligned} P_a &= p_a \\ P_b &= p_b \end{aligned} \tag{2.18}$$

Thus the mathematical derivation of the stress field for a large class of non-linear and plastic materials with finite strain has been reduced to the integration of an ordinary first order differential equation and a quadrature.

As an illustration let us examine the case of linear stress–strain relations. With coefficients α and β relations (2.6) become

$$\begin{aligned} \epsilon_1 &= \alpha\tau_1 + \beta\tau_2 \\ \epsilon_2 &= \alpha\tau_2 + \beta\tau_1 \end{aligned} \tag{2.19}$$

They satisfy condition (2.7) for isotropy. The differential equation (2.11) reduces to

$$\frac{d\tau_1}{d\tau_2} = -1 \tag{2.20}$$

Its general solution is

$$\tau_1 + \tau_2 = C \tag{2.21}$$

with an arbitrary constant C.

Note that the existence of a solution of the type (2.21) does not require the stress–strain relations to be linear. Consider for example the stress–strain relations

$$\begin{aligned} \epsilon_1 &= \alpha\tau_1 + \beta\tau_2 + f(\tau_1 + \tau_2) \\ \epsilon_2 &= \alpha\tau_2 + \beta\tau_1 + f(\tau_1 + \tau_2) \end{aligned} \tag{2.22}$$

where $f(\tau_1 + \tau_2)$ denotes an arbitrary function of $\tau_1 + \tau_2$. In this case we derive the same differential equation (2.20) and relation (2.21) is again valid.

It is also of interest to point out a general and important consequence of the assumption of isotropy which concerns the slope $d\tau_1/d\tau_2$ of the integral curves for points situated on the line $\tau_1 = \tau_2$. The property of isotropy requires that in the vicinity of the line $\tau_1 = \tau_2$ the differentials of stresses and strains be related by the following relations

$$\begin{aligned} d\epsilon_1 &= \alpha\, d\tau_1 + \beta\, d\tau_2 \\ d\epsilon_2 &= \alpha\, d\tau_2 + \beta\, d\tau_1 \end{aligned} \tag{2.23}$$

Hence

$$\frac{\partial\epsilon_2}{\partial\tau_2} = \alpha \quad \frac{\partial\epsilon_2}{\partial\tau_1} = \beta \tag{2.24}$$

Under the same conditions we may also write

$$\frac{\epsilon_1 - \epsilon_2}{\tau_1 - \tau_2} = \frac{d\epsilon_1 - d\epsilon_2}{d\tau_1 - d\tau_2} = \alpha - \beta \tag{2.25}$$

Substitution of these values into equation (2.11) shows

$$\frac{d\tau_1}{d\tau_2} = -1 \tag{2.26}$$

for $\tau_1 = \tau_2$. Hence for isotropic materials with arbitrary stress–strain relations the integral curves in the $\tau_1\tau_2$ plane are orthogonal to the line $\tau_1 = \tau_2$.

3. APPLICATION TO A CYLINDRICAL CAVITY

The rock material is of infinite extent with an infinitely long cylindrical cavity. A fluid pressure p is applied inside the cavity. The strain is assumed not to exceed a few per cent so that p may be identified approximately with a stress per unit initial area. At large distance the stress is assumed to be a uniform hydrostatic pressure σ. The effect of the rock porosity is neglected. This is justified if the fluid viscosity is sufficiently high or if the surface of the cavity is sealed.

Let us first analyze the case of a material governed by the linear stress–strain relation (2.19) and such that a crack occurs as the stress reaches the tensile value

$$\tau_2 = -R \tag{3.1}$$

The analysis is conveniently represented in a $\tau_1\tau_2$ diagram (Fig. 1). The integral of equation (2.11) in this case is a particular case of relation (2.21) namely

$$\tau_1 + \tau_2 = 2\sigma \tag{3.2}$$

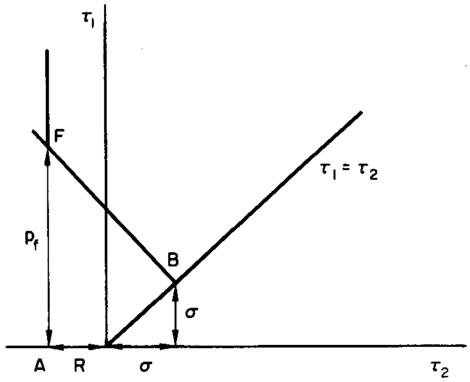

Fig. 1. Diagram $\tau_1\,\tau_2$ for linear material.

It is represented by the line BF in Fig. 1. It crosses the failure line AF at a point F whose ordinate $\tau_1 = p_f$ corresponds to incipient cracking. The failure pressure p_f is obtained by substituting $\tau_1 = p_f$ and $\tau_2 = -R$ in equation (3.2). Hence

$$p_f = R + 2\sigma \tag{3.3}$$

In a p_f vs. σ diagram this is represented by a straight line of slope two. (Fig. 2).

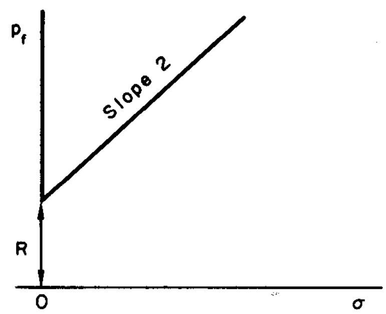

Fig. 2. Failure pressure p_f vs. hydrostatic stress σ for linear material.

Actually rock materials exhibit strongly non-linear properties. A typical stress–strain law for plane strain under a single stress τ_1, with $\tau_2 = 0$ is illustrated in Fig. 3 where

$$\begin{aligned} f_1(\tau_1) &= \epsilon_1(\tau_1, 0) \\ -f_2(\tau_1) &= \epsilon_2(\tau_1, 0) \end{aligned} \tag{3.4}$$

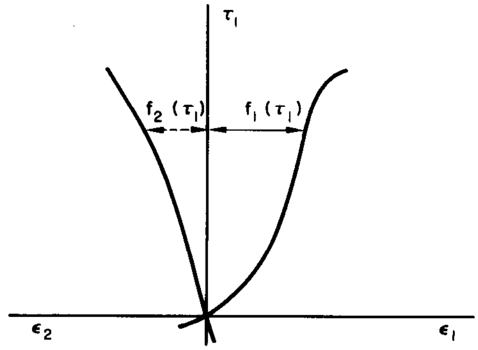

Fig. 3. Stress–strain functions corresponding to one type of non-linearity.

The shape of these functions is suggested by the experimental work of Brace [2].

Let us estimate typical solutions of the differential equation (2.11) in the $\tau_1 \tau_2$ plane. We have already shown that for any isotropic material the slope $d\tau_1/d\tau_2$ is minus one on the line $\tau_1 = \tau_2$. In order to bring out the general behavior of the integral curves it is sufficient to evaluate the slopes on the line $\tau_2 = 0$. On this line the slope (2.11) is written

$$\frac{d\tau_1}{d\tau_2} = \frac{[(\partial\epsilon_2)/(\partial\tau_2)](\tau_1, 0)}{(1/\tau_1)[f_1(\tau_1) + f_2(\tau_1)] - (df_2/d\tau_1)} \tag{3.5}$$

If we examine the physical significance of the terms on the right side we see that we may write approx.

$$\frac{f_2}{\tau_1} = \frac{df_2}{d\tau_1} \tag{3.6}$$

at least for values of τ_1 which are not too high. On the other hand the quantity $\partial\epsilon_2(\tau_1, 0)/\partial\tau_2$ represents a compliance for the transversal strain ϵ_2 and for small values of τ_2. Due to isotropy, this compliance, in the vicinity of $\tau_1 = 0$, is

$$\frac{\partial\epsilon_2}{\partial\tau_2}(0, 0) = \frac{\partial\epsilon_1}{\partial\tau_1}(0, 0) = \frac{df_1}{d\tau_1}(0) = f_1'(0) \tag{3.7}$$

For values of τ_1 which are not too large this compliance should decrease only slightly so that we may write approx.

$$\frac{\partial\epsilon_2}{\partial\tau_2}(\tau_1, 0) = f_1'(0) \tag{3.8}$$

Hence with reasonable accuracy the slope (2.11) on the line $\tau_2 = 0$ becomes

$$\frac{d\tau_1}{d\tau_2} = -\frac{f_1'(0)}{(1/\tau_1)f_1(\tau_1)} \tag{3.9}$$

As can be seen this is given by the ratio of two slopes on the curve $f_1(\tau_1)$, the slope at the origin, and the slope of the chord from the origin to the point of ordinate τ_1. For the curve $f_1(\tau_1)$ shown in Fig. 3 the absolute value of this ratio is larger than unity, hence

$$\left|\frac{d\tau_1}{d\tau_2}\right| > 1 \tag{3.10}$$

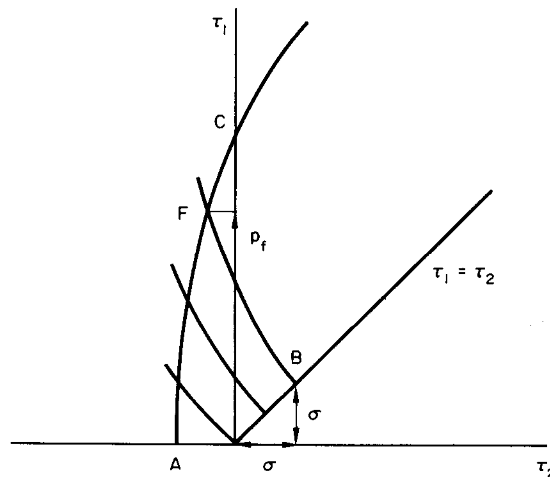

Fig. 4. Diagram $\tau_1 \tau_2$ for non-linearity illustrated in Fig. 3.

As a consequence the integral curves in the $\tau_1 \tau_2$ plane are concave upward as illustrated in Fig. 4. Crack formation will generally be represented by a failure line of the type *AFC*. The point *F* at which the integral *BF* crosses the failure line yields the cavity pressure p_f for incipient cracking. The plot of p_f vs. σ derived from Fig. 4 is shown in Fig. 5.

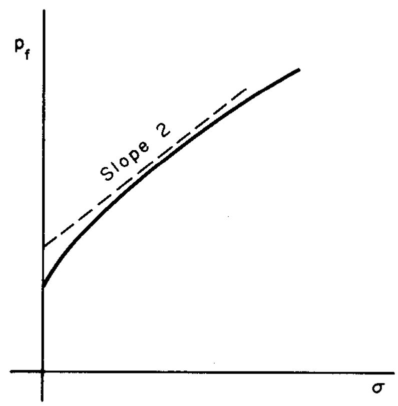

Fig. 5. Failure pressure p_f vs. hydrostatic stress σ for non-linearity illustrated in Fig. 3.

The slope $dp_f/d\sigma$ will be about two or may be larger than two for σ small and decrease for larger values of σ. This result is a consequence of the type of non-linearity represented in Fig. 3. It is to be expected for materials where elastic non-linearity predominates. In general this will be the case when the non-linearity is due to closing of the pores.

Depending on the shape of the failure line *AFC* and of the stress–strain functions $f_1(\tau_1)$ and $f_2(\tau_2)$ the plots of p_f vs. σ will exhibit a variety of characteristics. For example consider the material with the type of stress–strain functions shown in Fig. 6 and the type of failure line shown in Fig. 7. This type of behavior will generally be associated with materials for which plastic properties predominate. The integral curves in the plane $\tau_1 \tau_2$ are illustrated in Fig. 7. They will be convex upward. The resulting plot of failure pressure p_f vs. the hydrostatic pressure σ is illustrated in Fig. 8. The slope is closer to unity and the curve may have an inflexion point.

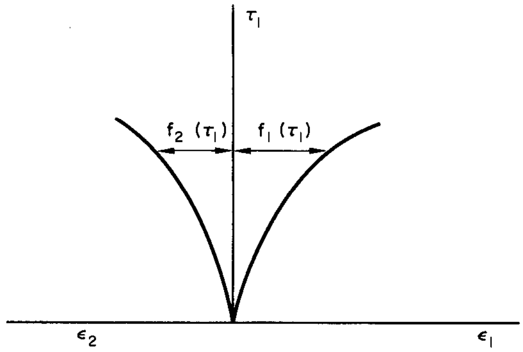

Fig. 6. Stress–strain functions corresponding to another type of non-linearity.

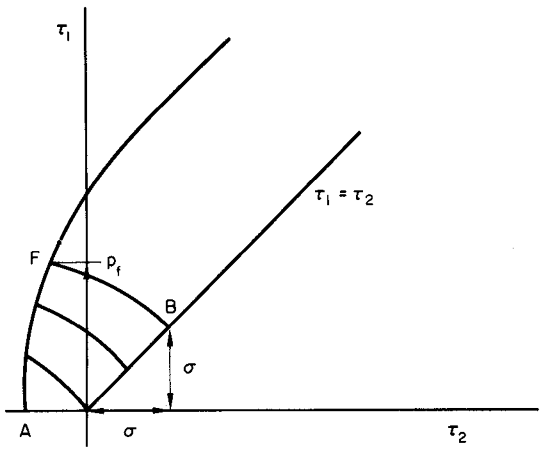

Fig. 7. Diagram $\tau_1\ \tau_2$ for non-linearity illustrated in Fig. 6.

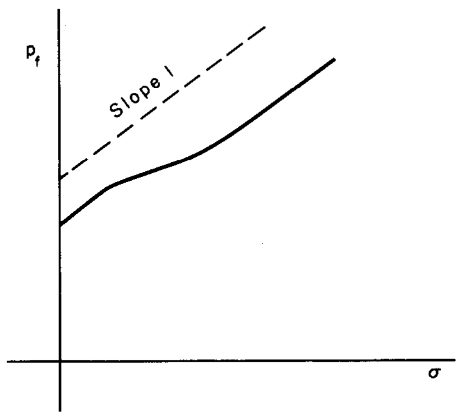

Fig. 8. Failure pressure p_f vs. hydrostatic stress σ for non-linearity illustrated in Fig. 6.

4. STRESS AROUND A SPHERICAL CAVITY

Spherical symmetry of the stress and displacement fields is assumed. A material point initially at a distance r from the center is displaced radially by an amount U function of r only. The radial stress at the displaced point is denoted by τ_1. The circumferential stress τ_2 at this point is the same in all directions normal to the radius r. As before these principal stresses represent forces per unit initial area. The corresponding principal strain components are formally the same as expressions (2.1). The radial component ϵ_1 and the circumferential component ϵ_2 are respectively

$$\epsilon_1 = -\frac{dU}{dr} \qquad \epsilon_2 = -\frac{U}{r} \tag{4.1}$$

The principal of virtual work in this case is written

$$\pi \int (\tau_1 \delta\epsilon_1 + 2\tau_2 \delta\epsilon_2) r^2 \, dr = 0 \tag{4.2}$$

Substitution of expression (4.1) for the strain and integration by parts yield the equilibrium condition for the stress field

$$\frac{d\tau_1}{dr} + \frac{2(\tau_1 - \tau_2)}{r} = 0 \tag{4.3}$$

The compatibility condition for the strain is the same as (2.5)

$$\frac{d\epsilon_2}{dr} = \frac{\epsilon_1 - \epsilon_2}{r} \tag{4.4}$$

Elimination of r between equations (4.3) and (4.4) leads to the relation

$$\frac{d\tau_1}{d\epsilon_2} = -2\frac{\tau_1 - \tau_2}{\epsilon_1 - \epsilon_2} \tag{4.5}$$

We now examine the stress–strain relations. For an isotropic material the three principal stress $\tau_1 \tau_2 \tau_3$ and the corresponding finite strain components $\epsilon_1 \epsilon_2 \epsilon_3$ are related by the equations

$$\begin{aligned} \epsilon_1 &= \varphi(\tau_1, \tau_2, \tau_3) \\ \epsilon_2 &= \varphi(\tau_2, \tau_3, \tau_1) \\ \epsilon_3 &= \varphi(\tau_3, \tau_1, \tau_2) \end{aligned} \tag{4.6}$$

The property of isotropy also requires that the function φ satisfies the identity

$$\varphi(\tau_1, \tau_2, \tau_3) = \varphi(\tau_1, \tau_3, \tau_2) \tag{4.7}$$

In the present case, with spherical symmetry $\tau_2 = \tau_3$, $\epsilon_2 = \epsilon_3$, and the stress–strain relations become

$$\begin{aligned} \epsilon_1 &= \epsilon_1(\tau_1, \tau_2) = \varphi(\tau_1, \tau_2, \tau_2) \\ \epsilon_2 &= \epsilon_2(\tau_1, \tau_2) = \varphi(\tau_2, \tau_2, \tau_1) \end{aligned} \tag{4.8}$$

Note that for an elastic material the expression

$$\begin{aligned} \tau_1 \delta\epsilon_1 + 2\tau_2 \delta\epsilon_2 - \delta(\tau_1\epsilon_1 + 2\tau_2\epsilon_2) \\ = -\epsilon_1 \delta\tau_1 - 2\epsilon_2 \delta\tau_2 \end{aligned} \tag{4.9}$$

is an exact differential. Hence in this case the function (4.8) must also satisfy the condition

$$\frac{\partial}{\partial \tau_2}\epsilon_1(\tau_1, \tau_2) = 2\frac{\partial}{\partial \tau_1}\epsilon_2(\tau_1, \tau_2) \tag{4.10}$$

However this condition is not required for the validity of the present analysis.

From the stress–strain relations (4.8) we derive

$$d\epsilon_2 = \frac{\partial\epsilon_2}{\partial\tau_1} d\tau_1 + \frac{\partial\epsilon_2}{\partial\tau_2} d\tau_2 \qquad (4.11)$$

Substitution in equation (4.5) yields

$$\frac{d\tau_1}{d\tau_2} = -\frac{2[(\partial\epsilon_2)/(\partial\tau_2)]}{[(\epsilon_1 - \epsilon_2)/(\tau_1 - \tau_2)] + 2[(\partial\epsilon_2)/(\partial\tau_1)]} \qquad (4.12)$$

In the plane τ_1, τ_2 this is a first order differential equation for τ_1 as a function of τ_2.

Let us examine the case of linear stress–strain relations. The function φ is linear

$$\varphi(\tau_1, \tau_2, \tau_3) = \alpha\tau_1 + \beta(\tau_2 + \tau_3) \qquad (4.13)$$

hence

$$\begin{aligned} \epsilon_1(\tau_1, \tau_2) &= \alpha\tau_1 + 2\beta\tau_2 \\ \epsilon_2(\tau_1, \tau_2) &= \beta\tau_1 + (\alpha + \beta)\tau_2 \end{aligned} \qquad (4.14)$$

With the values (4.14) the differential equation (4.12) becomes

$$\frac{d\tau_1}{d\tau_2} = -2 \qquad (4.15)$$

The integral solution which goes through the point $\tau_1 = \tau_2 = \sigma$ is

$$\tau_1 + 2\tau_2 = 3\sigma \qquad (4.16)$$

This leads to a diagram similar to Fig. 1 except that the slope of the line BF is now minus two. For a medium with hydrostatic stress σ at large distance from the cavity, crack initiation occurs for a cavity pressure p_f given by

$$p_f = 2R + 3\sigma \qquad (4.17)$$

where $\tau_2 = -R$ is the biaxial tensile fracture stress assumed independent of τ_1. The plot of p_f vs. σ is analogous to that shown in Fig. 2 except that the slope of the line is now three instead of two.

For a non-linear material we proceed as previously for the cylindrical case. Consider the slope $d\tau_1/d\tau_2$ on the line $\tau_1 = \tau_2$. In the vicinity of the points $\tau_1 = \tau_2$ and $\epsilon_1 = \epsilon_2$ we replace the variable by their differentials. For these variables the material is isotropic. Hence the relations between the differentials must be of the same form as (4.14) namely,

$$\begin{aligned} d\epsilon_1 &= \alpha\, d\tau_1 + 2\beta\, d\tau_2 \\ d\epsilon_2 &= \beta\, d\tau_1 + (\alpha + \beta)\, d\tau_2 \end{aligned} \qquad (4.18)$$

where α and β are functions of $\tau_1 = \tau_2$. We also write

$$\begin{aligned} \epsilon_1 - \epsilon_2 &= d\epsilon_1 - d\epsilon_2 \\ \tau_1 - \tau_2 &= d\tau_1 - d\tau_2 \end{aligned} \qquad (4.19)$$

Substitution of these values into equation (4.12) yields

$$\frac{d\tau_1}{d\tau_2} = -2 \qquad (4.20)$$

on the line $\tau_1 = \tau_2$. Note that this is a consequence only of the property of isotropy. Hence in the plane τ_1, τ_2 the integrals of equation (4.12) initiated from the line $\tau_1 = \tau_2$ always start with a slope minus two and are concave upward or downward depending on the stress–strain relations. They will be similar to the curves in Figs. 4 and 7 for the cylindrical cavity.

For the plot of the failure pressure p_f vs. the hydrostatic stress σ results similar to those of Figs. 5 and 8 are obtained, except that the slopes will tend to be higher in the ratio of about three to two.

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